MATH 4030 Differential Geometry
Tutorial 2, 20 September 2017

1. Find T, N, B, κ, τ of the helix $\alpha(s) = \left(\cos\frac{s}{\sqrt{2}}, \sin\frac{s}{\sqrt{2}}, \frac{s}{\sqrt{2}}\right)$, $s \in \mathbb{R}$.

   - We skip the step of arc length reparametrization because α is already in that form.
   - $T = \alpha'(s) = \left(-\frac{1}{\sqrt{2}}\sin\frac{s}{\sqrt{2}}, \frac{1}{\sqrt{2}}\cos\frac{s}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
   - $T' = \left(-\frac{1}{2}\cos\frac{s}{\sqrt{2}}, -\frac{1}{2}\sin\frac{s}{\sqrt{2}}, 0\right)$
   - $\kappa = |T'| = \frac{1}{2} \neq 0$
   - $N = \frac{T'}{\kappa} = \left(-\cos\frac{s}{\sqrt{2}}, -\sin\frac{s}{\sqrt{2}}, 0\right)$
   - $B = T \times N = \det\begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -\frac{1}{\sqrt{2}}\sin\frac{s}{\sqrt{2}} & \frac{1}{\sqrt{2}}\cos\frac{s}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\cos\frac{s}{\sqrt{2}} & -\sin\frac{s}{\sqrt{2}} & 0 \end{pmatrix} = \left(\frac{1}{\sqrt{2}}\sin\frac{s}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\cos\frac{s}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
   - $B' = \left(\frac{1}{2}\cos\frac{s}{\sqrt{2}}, \frac{1}{2}\sin\frac{s}{\sqrt{2}}, 0\right)$
   - $\tau = \langle B', N\rangle = -\frac{1}{2}$.

2. (HW 1 Suggested problem Q10) By considering a rigid motion of $\mathbb{R}^2$, we assume $p = \mathbf{0}$. For each $t \in I$, if $\alpha(t) \neq \mathbf{0}$, then from the assumption we see that $\alpha(t)$ is parallel to the normal line to α at $\alpha(t)$, and hence that $\langle \alpha(t), \alpha'(t)\rangle = 0$. If, on the other hand, $\alpha(t) = \mathbf{0}$, then we also have $\langle \alpha(t), \alpha'(t)\rangle = 0$. Hence by HW 1 Suggested problem Q8, we conclude that $|\alpha(t)| = r$ which is a positive constant. Geometrically speaking, α lies in the circle with center p and radius $r > 0$.

3. Read do Carmo's book p.27-30